Pricing Unit-Linked Insurance Contracts using Estimated Volatility

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Abstract

This paper develops a model for pricing a unit-linked insurance contract by estimating the volatility. This insurance contract with minimum death guarantee is a contingent claim which implies that a hedging argument can be used to determine the price. In this case, the guarantee strike price does not depend on the current time and the insurer's liability for a death at a given time is similar to the terminal cash flow of a European put option and we end up with a Black-Scholes like put pricing formula. In this paper, we extend the work of Frantz et al. (2003) by relaxing the assumption that volatility is constant.

Keywords: unit-linked insurance contract, premiums, guaranteed minimum death benefit

1. Introduction

The fair valuation of a unit-linked insurance is only valid if hedging is applied to the underlying financial asset (Frantz et al. 2003). In such contracts, the return obtained by the policyholders on their savings is linked to some financial asset, and in this way it is the policyholder who supports the risk of the investment. The investment can be made on one asset or on a portfolio of assets.

The first theoretical analysis of this problem dates back to (Brennan & Schwartz 1976) and (Boyles & Schwartz 1977). In these papers, they apply the Black-Scholes option pricing methodology to the pricing of guarantees in equity-linked life insurance policies. The underlying assumption is the completeness of the market. Assuming that the financial market is complete, this implies that the insurer is "risk-neutral" with respect to the mortality risk. This assumption is only approximately true for extremely large portfolios.

In this paper, the insurer's liability for a death at time t will be:

$$\max(K, S_t) = S_t + \max(K - S_t, 0)$$

where:

$\max(K - S_t, 0)$ corresponds to the sum at risk and is similar to the terminal cash-flow of a European put option with strike price K. (Frantz et al. 2003) analyzed delta hedging within the framework of (Black & Scholes model 1973). Black and Scholes model supposes that the returns process is continuous, distributed according to a normal distribution, and that its volatility is constant during time. However, the empirical reports show that none of these assumptions are always true when applied to the markets, as shown by the works of (Cont 2001).

A unit linked fund also referred to as an insured fund is a fund that is linked to a plan issued by an insurance company that allows you to combine your money with other plan holders and hence invest in a much wider spread of investments than if you were to invest on your own. As an investor you combine your money with money from other investors and buys units in a fund. A unit linked fund is divided into units of equal value. The value, or price, of each unit depends on the value of the assets of the unit linked fund, whereas unit price determines the number of units you receive when you invest money in the fund, and the sum you receive when you sell your units.

Units represent the way that a fund is split to identify a value that we can attribute to each customers holding. The accumulation of these premiums are thus reinvested and consequently earn interest depending on performance of the market. You can also make lump sum contributions depending on the fund. Just like any other investment plan, different funds will have different levels of risk. For example, corporate bonds tend to have a higher level of risk while compared to government bonds.

Due to the financial instability caused by the unexpected market movements and the amplification of market competitiveness, insurers from international markets have started to incorporate guarantees in unit-linked products. Investment guarantees are very popular features in life insurance policies because in addition to paying a death benefit or a maturity benefit, these policies are tied to the return of an underlying asset or an actively managed portfolio. Thus, the policy also acts as an investment because the investor's capital is credited with a minimum return. In exchange for this protection, the policyholder pays a higher premium, reflecting the market risk assumed by the insurance company.

An important component of the activity carried out by the insurance companies is the investment of the premiums paid by policyholders in various types of assets, in order to obtain higher yields than those guaranteed by the insurance contracts, while providing the necessary liquidity for the payment of insurance claims in case of occurrence of the assumed risks. So the guaranteed benefits can be broadly matched or immunized with various types of financial assets, especially with fixed-interest instruments (Hardy, 2002)

The introduction of unit linked insurance plans has possibly been the single largest innovation in the field of life insurance. It has addressed and overcome many difficulties and concern that customers had about life insurance in terms of liquidity, flexibility, and transparency. These benefits are possible because unit linked insurance plans are differently structured products and leave many choices to the policyholder. They are structured such that the protection (insurance) element and the savings element (investment) can be distinguish and hence managed according to one's specific needs, offering flexibility and transparency.

Unit linked funds are all set to pose serious competition to mutual funds. Though as an investment avenue they are closest to mutual funds in terms of their structure and functioning, unit linked funds are essentially a long term commitment between the policyholder and the insurance company and mutual funds are built to cater to the relatively short –term need of the investor. The investments are made with a shorter- term duration profile when compared to Unit linked funds. The seemingly similar structure of both of them makes it vital for investors to be aware of the fine distinctions in both the offering and make informed decisions.

In the United Kingdom, unit-linked insurance rose in popularity in the late 1960s. In the United States, variable life insurance started in the 1980s, while in Canada unit-linked fund contracts became popular in the late 1990s. The functions of unit-linked insurance are to provide the insured both the life protection and investment opportunity. The insurer invests the premiums received less deductions in a separate fund.

2. Pricing

The pricing formula for a put option is shown below;

$$P(S,T,K,r,\sigma) = Ke^{-r(T-t)}N(-d_2) - SN(-d_1) \qquad (1)$$

$$d_1(t,T) = \frac{\ln\left(\frac{S}{K}\right) + \left(r + \frac{\sigma^2}{2}\right)(T-t)}{\sigma\sqrt{T-t}}, \qquad d_2(t,T) = d_1 - \sigma\sqrt{T-t}$$

Here;

N is the cumulative standard normal distribution function,

r is the current risk-free rate of return, S is the current stock price,

K is the strike price,

T-t is the time until expiration in years,

and σ is the (annualized) volatility of the stock.

According to Gross (2006), an estimate of the volatility will be be obtained from the equation;

$$\hat{\sigma} = \frac{\sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(\mu_i - \bar{\mu})^2}}{\sqrt{T-t}} \qquad (2)$$

where $\mu_i = ln\left(\frac{S_i}{S_{i-1}}\right)$, $\bar{\mu}$ is the sample average of all μ_i, S_i is the stock price in period i. The numerator is readily

seen to be the sample standard deviations of the μ_i, the log-returns of the stock, and the denominator is a scaling

factor to make the estimate be one of yearly volatility.

Given **Error! Reference source not found.** we can now evaluate the expected loss in t for a death in T. Thus we define:

$$V(t,T) = E_Q\left[e^{-r(T-t)}\max(K - S_t, 0)/\mathcal{F}_t\right]$$

where the filtration $\mathcal{F}_t$ represents all the information generated by the evolution of the asset price and the mortality up to time t.

We obtain an expression for the single pure premium *SPP* as given below;

$$SPP = \sum_{k=1}^{\omega-x} V(0,k) \, {}_kp_x q_{x+k}$$

$$= K \sum_{k=1}^{T-t} e^{-rk} N(-d_2(0,k)) \, {}_kp_x q_{x+k} - S \sum_{k=1}^{T-t} N(-d_1(0,k)) \, {}_kp_x q_{x+k}$$

$$d_1(t,T) = \frac{ln\left(\frac{S}{K}\right) + \left(r + \frac{\hat{\sigma}^2}{2}\right)(T-t)}{\hat{\sigma}\sqrt{T-t}}, \quad d_2(t,T) = d_1 - \hat{\sigma}\sqrt{T-t}$$

The premium is a sum of Black-Scholes put prices. The risk-free rate *r* is replaced by the expected return μ.

The premium paid is divided into unit world and cash or non-unit world. The policyholder pays premiums to acquire units, and the eventual benefit is normally denominated in these units, so we will need to keep track of the number of units bought by a policyholder, how they are growing, and what charges we are deducting from them. However, the policyholder pays the life insurance company in real money. So we need to keep track of the cash not used to buy units, because that cash is a source of profit to the life insurance company. Conversely, if the policyholder dies there might be a cash denominated sum insured, and so we need to keep track of the cash outgo on claims. Another very significant cash outgo to consider is comprised of the company's expenses. These will include expenses incurred in underwriting and maintaining the policy, as well as commission payments to whoever sold it.

3. Stochastic simulation methods

Boyle and Hardy (1997), dealt with maturity guarantee, which provide a minimum level of benefits at contact maturity. When the contract matures, the value of the benefit is guaranteed not to fall below a certain level. They used two approaches for pricing and reserving for these guarantees; stochastic simulation of future investment returns and modern option pricing theory.

The first approach, which we will consider in our case, uses a stochastic model of future investment experience to generate a number of scenarios of stock price movements over the term of the fund policy. This simulations can be used to generate a sample of the accumulated premiums at maturity, using the random and equally likely projected investment conditions. From this they estimated the distribution of the claims under the guarantee. The distribution is then used to estimate quantities of interest such as the expected cost of the guarantee and the amount of the initial reserve such that the provision will be adequate say 99% of the time. To determine the appropriate rate to discount the future claims need to make an assumption concerning the investment of the reserves.

Boyle and Hardy (1997), had weakness since if the investment performance is poor in the last year or two of the simulation, then the claim can be significant. This is because the distribution of the claims under the guarantee

will generally be very highly skewed. Most of the time there will be no cost under the guarantee as the accumulated premiums will exceed the guarantee.

4. Impacts of guaranteed minimum death benefit on profit margins of Unit Linked Funds

Changki (2008), did a research on the impacts of the embedded guaranteed minimum death benefit (GMDB) options on the profit margin of the unit linked policies. He argued that the profits from the policies are directly linked to the non-unit fund account but also depend on the cash flows in the unit account , which suggests that the insurers are able to make profits for themselves through investing a portion of the premiums received from the policyholders

For insurance companies which are likely to provide GMDB to the policyholders, they can take a short position on a put option which has the payoff of the excess death benefit over the unit account value if it is positive or otherwise. Hence the insurers are likely to charge a premium of such a synthetic put option to compensate the risk of cash outflow.

He showed that the profit margins without GMDB options are more than those with GMDB options. For the policies embedded with GMDB options but not charging any option prices the profit margins are quite low which can a burden to the issuing companies

5. Conclusion

In conclusion, each policyholder receives the value of the units allocated to the policy. There is no pooling of investments or allocation of the pooled surplus. As each premium is paid, a specified proportion (the "allocation percentage") is invested in an investment fund chosen by the policyholder. The investment fund is divided into units which are priced continuously. When each investment allocation is made, the number of units purchased by the policyholder is recorded.

The value at the date of death or survival (i.e. at the time of the claim) of the cumulative number of units purchased is the sum assured under the policy. Sometimes a minimum guaranteed sum assured is specified in the terms of the contract to ensure that the policyholder avoids any difficulties arising from a particularly poor investment performance. In addition, a minimum guaranteed sum assured will give the policyholder (or his estate) some benefit in the event of an early death. Without it the policy becomes a pure savings policy, with no element of life insurance involved

Unit-linked policies can have a minimum guaranteed sum assured payable on survival to a specified date, on death, or both. Death benefit guarantees are generally more common than survival (or maturity) guarantees. In order to price and value unit-linked contracts, details of allocation percentages (usually specified in the policy) and an assumption about the future growth in the price of the units purchased are needed.

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